UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 115A
Prof. Rollin J. Morrison

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Homework #5

Midterm

Answers

Information : $\hbar = 1 \times 10^{-34}$ joules/sec
$m_e = 9 \times 10^{-31}$ kg
$e = 1.6 \times 10^{-19}$ coulombs

1. (20 points) An electron is bound in a region of space about the size of an atom, 10^{-10} m. Make a very rough estimate of the minimum kinetic energy which the electron must have.

$\Delta p \Delta x \sim \hbar \qquad p \sim \Delta p = \frac{\hbar}{\Delta x} = \frac{\hbar}{a} \qquad a = 10^{-10}\,m$

$KE = \frac{p^2}{2m} = \left(\frac{\hbar}{a}\right)^2 \frac{1}{2m} = \left(\frac{10^{-34}}{10^{-10}}\right)^2 \frac{J^2 sec^2}{2 \times 9 \times 10^{-31}\, m^2 kg}$

$J = \frac{kg\, m^2}{sec^2}$

$KE = \frac{1}{2} 10^{-18}\, J$

2. (30 points) A particle of mass m is bound in an infinite one-dimensional square well where $V = \infty$ for $x < 0$ and $x > a$, and $V = 0$ for $0 < x < a$. The energy eigenfunctions for this square well are $\phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$, corresponding to the energies $E_n = \frac{(\hbar \frac{n\pi}{a})^2}{2m}$. Consider the state at $t = 0$, $|\psi(0)\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$.

(a) Is this a stationary state?

$\psi^* \psi = \frac{1}{2}\left[e^{-\frac{i}{\hbar}E_1 t}\phi_1 + e^{-\frac{i}{\hbar}E_2 t}\phi_2\right]^* \left[e^{-\frac{i}{\hbar}E_1 t}\phi_1 + e^{-\frac{i}{\hbar}E_2 t}\phi_2\right]$

$= \frac{1}{2}\left[\phi_1^*\phi_1 + \phi_2^*\phi_2 + \phi_1^*\phi_2 e^{-\frac{i}{\hbar}(E_2 - E_1)t} + \phi_1\phi_2^* e^{\frac{i}{\hbar}(E_2 - E_1)t}\right]$

Since $P(x) = \psi^*\psi$ depends upon time ψ is not a stationary state. Each ϕ_n is a stationary state, but not ψ.