4. (4.5) To find the energy eigenstates, use Eq'ns (4-47) and (4-48) w/ $a \to 2a$ for the initial well states or $a \to 2b$ for the final well states. Let ψ_n^a be energy eigenstates for 1st well and ψ_n^b be energy eigenstates for 2nd well. Then:

Amplitude for ψ_1^a (ground state of 1st well) to be in ψ_1^b (ground state of 2nd well) $\equiv A_{11}$

$$A_{11} = \int_{-\infty}^{\infty} dx\, \psi_1^{b*}\, \psi_1^a = \int_{-a}^{a} dx \left[\frac{1}{\sqrt{b}} \cos\left(\frac{\pi x}{2b}\right)\right]\left[\frac{1}{\sqrt{a}} \cos\left(\frac{\pi x}{2a}\right)\right]$$

† since ψ_1^a is 0 outside this range

$$= \frac{1}{\sqrt{ab}} \frac{4ab^2}{\pi(b^2-a^2)} \cos\left(\frac{\pi}{2}\frac{a}{b}\right) \quad \text{by Mathematica}$$

$P_{11} \equiv$ Probability that ψ_1^a is in $\psi_1^b = |A_{11}|^2$

$$\boxed{P_{11} = \left(\frac{4}{\pi}\right)^2 \left(\frac{a}{b}\right)\left(1 - a^2/b^2\right)^{-2} \cos^2\left(\frac{\pi}{2}\frac{a}{b}\right)}$$

$$A_{12} = \int_{-\infty}^{\infty} dx\, \psi_2^{b*}\, \psi_1^a = \int_{-a}^{a} dx \left[\frac{1}{\sqrt{b}} \sin\left(\frac{\pi x}{b}\right)\right]\left[\frac{1}{\sqrt{a}} \cos\left(\frac{\pi x}{2a}\right)\right] = 0$$

So $\boxed{P_{12} = |A_{12}|^2 = 0}$

The reason that $P_{12}=0$ is that ψ_2^b and ψ_1^a have opposite parity.

$$\psi_1^a = \sum_n A_n \psi_n^b = \sum_{n\ \text{odd}} A_n \psi_n^b + \sum_{n\ \text{even}} A_n \psi_n^b \quad \text{in general}$$

But since ψ_1^a is even it can be expanded only in terms of even functions. If the ψ_n^b's didn't have definite parity this wouldn't help, but since ψ_n^b is even for n odd and odd for n even, we see that A_n for n even $= 0$. So we have $\psi_1^a = \sum_{n\ \text{odd}} A_n \psi_n^b$. In particular $A_2 = 0 \Rightarrow A_{12} = 0$