1 (cont.) $\langle p \rangle = \int dx\ \Psi^*(x,t)\left(-i\hbar \frac{\partial}{\partial x}\right)\Psi(x,t)$

$$= -i\hbar\ \frac{1}{2} \int_{-a/2}^{a/2} dx \left[\psi_0^*(x) e^{i\frac{E_0}{\hbar}t} + \psi_1^*(x) e^{i\frac{E_1}{\hbar}t}\right]\left[\psi_0'(x) e^{-i\frac{E_0}{\hbar}t} + \psi_1'(x) e^{-i\frac{E_1}{\hbar}t}\right]$$

where $\psi_n'(x) = \frac{\partial}{\partial x}\psi_n(x)$

$$= -i\hbar\ \frac{1}{2} \int dx \left\{ \underbrace{\psi_0^*(x)\psi_0'(x)}_{\text{odd}} + \underbrace{\psi_1^*(x)\psi_1'(x)}_{\text{odd}} + \underbrace{\psi_0^*(x)\psi_1'(x)}_{\text{even}} e^{-i\frac{t}{\hbar}(E_1-E_0)} + \underbrace{\psi_1^*(x)\psi_0'(x)}_{\text{even}} e^{i\frac{t}{\hbar}(E_1-E_0)} \right\}$$

So 1st 2 terms integrate to 0. To see evenness and oddness consider:

$\psi_1 \sim \sin x \Rightarrow \psi_1' \sim \cos x$ (odd) (even) and $\psi_0 \sim \cos x \Rightarrow \psi_0' \sim \sin x$ (even) (odd)

and also, $\psi_n^* = \psi_n$ -- eigenfunctions real

$$= -i\hbar\ \frac{1}{2} \int_{-a/2}^{a/2} dx \left\{ \left(\sqrt{\tfrac{2}{a}}\cos\left(\tfrac{\pi x}{a}\right)\right)\left(\tfrac{2\pi}{a}\sqrt{\tfrac{2}{a}}\cos\left(\tfrac{2\pi x}{a}\right)\right) e^{-i\frac{t}{\hbar}(E_1-E_0)} \right.$$

$$\left. + \left(\sqrt{\tfrac{2}{a}}\sin\left(\tfrac{2\pi x}{a}\right)\right)\left(-\tfrac{\pi}{a}\sqrt{\tfrac{2}{a}}\sin\left(\tfrac{\pi x}{a}\right)\right) e^{i\frac{t}{\hbar}(E_1-E_0)} \right\}$$

$$= -i\hbar\ \frac{1}{2}\ \frac{2}{a}\ \frac{\pi}{a} \int_{-a/2}^{a/2} dx \left\{ 2\cos\left(\tfrac{\pi x}{a}\right)\cos\left(\tfrac{2\pi x}{a}\right) e^{-i\frac{t}{\hbar}(E_1-E_0)} - \sin\left(\tfrac{\pi x}{a}\right)\sin\left(\tfrac{2\pi x}{a}\right) e^{i\frac{t}{\hbar}(E_1-E_0)} \right\}$$

$$= -i\hbar\ \frac{1}{2}\ \frac{2\pi}{a^2} \left[ \frac{4}{3}\ \frac{a}{\pi}\ e^{-i\frac{t}{\hbar}(E_1-E_0)} - \frac{4}{3}\ \frac{a}{\pi}\ e^{i\frac{t}{\hbar}(E_1-E_0)} \right] \quad \text{by mathematica}$$

$$= -\frac{8}{3}\left(\frac{a}{\hbar}\right)^{-1} \sin\left[\frac{1}{\hbar}(E_1-E_0)t\right]$$

$$\boxed{\langle p \rangle = -\frac{8}{3}\left(\frac{a}{\hbar}\right)^{-1} \sin\left[\frac{3}{2}\ \frac{\hbar\pi^2}{ma^2}\ t\right]}$$

So we see $\langle x \rangle$ and $\langle p \rangle$ are oscillating 90° out of phase. The "particle" is bouncing back and forth in the box, but it never gets to the walls -

$$-\left(\frac{\sqrt{2}4}{3\ \pi}\right)^2 \frac{a}{2} < \langle x \rangle < \left(\frac{4\sqrt{2}}{3\ \pi}\right)^2 \frac{a}{2} \qquad \text{(The walls are at } \pm a/2\text{)}$$

Note also that $m\frac{d}{dt}\langle x \rangle = \langle p \rangle$

In the appropriate classical limit, we expect $\langle x \rangle$, $\langle p \rangle$ to become the position, momentum of the particle.