1. $\Psi(x,t) = \frac{1}{\sqrt{2}}[\psi_0(x,t) + \psi_1(x,t)]$

Eqtn (4-33) $\langle H\rangle = \langle E\rangle = \sum E_n |A_n|^2 = E_0\left|\frac{1}{\sqrt{2}}\right|^2 + E_1\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}(E_0+E_1)$

$\therefore \langle E\rangle = \frac{1}{2}\frac{\hbar^2\pi^2}{2ma^2}(1+4) = \boxed{\frac{5}{4}\frac{\hbar^2\pi^2}{ma^2} = \langle E\rangle}$

$\langle E\rangle$ has no time dependence. You can see why this happens by inserting the time dependence back into the 3rd step of Eqtn. (4-33). The integral being done is $\int dx\, \Psi^* u_n(x) = \int dx \left\{\sum_m A_m^* u_m^*(x) e^{iE_m t/\hbar}\right\}\{u_n(x) e^{-iE_n t/\hbar}\}$

$= \sum_m A_m^* e^{it/\hbar(E_m - E_n)} \underbrace{\int dx\, u_m^* u_n}_{\delta_{mn}} = A_n^*$

Time dependence can only come from "off diagonal" ($m \neq n$) terms, but for these terms the integral $\to 0$ since eigenvectors orthonormal.

$\langle x\rangle = \int dx\, \Psi^*(x,t)\, x\, \Psi(x,t) = \frac{1}{2}\int dx \left[\psi_0^*(x)e^{i\frac{E_0}{\hbar}t} + \psi_1^*(x)e^{i\frac{E_1}{\hbar}t}\right] x \left[\psi_0(x)e^{-i\frac{E_0}{\hbar}t} + \psi_1(x)e^{-i\frac{E_1}{\hbar}t}\right]$

$= \frac{1}{2}\int dx\, x\left[\psi_0^*(x)\psi_0(x) + \psi_1^*(x)\psi_1(x) + \psi_0^*(x)\psi_1(x)e^{\frac{it}{\hbar}(E_0-E_1)} + \psi_1^*(x)\psi_0(x)e^{\frac{it}{\hbar}(E_1-E_0)}\right]$

But for square well, $\psi_n^*(x) = \psi_n(x)$, so:

$= \frac{1}{2}\int_{-a/2}^{a/2} dx\, \underbrace{x}_{\text{odd}}\left[\underbrace{\psi_0^2(x)}_{\text{even}} + \underbrace{\psi_1^2(x)}_{\text{even}} + \underbrace{\psi_0\psi_1}_{\text{odd}}\left(e^{i\frac{5}{\hbar}(E_1-E_0)} + e^{-i\frac{5}{\hbar}(E_1-E_0)}\right)\right]$

So we see that the 1st 2 terms will integrate to zero — the integral of an odd fcn w/ an even intrval is 0. You can see the even-ness or odd-ness of terms in [ ]'s either by substituting in $\psi_0 \sim \cos x$, $\psi_1 \sim \sin x$ and looking at fcns, or equivilantly, you know ψ_n's have definite parity & sequential levels alternate so ψ_n^2 even and $\psi_n\psi_{n+1}$ odd.

$= \cos\left[\frac{1}{\hbar}(E_1-E_0)t\right]\int_{-a/2}^{a/2} dx\, x\left(\sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)\right)\left(\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right)\right)$ (Using 4-47 & 4-48)

$= \cos\left[\frac{1}{\hbar}(E_1-E_0)t\right]\frac{2}{a}\left\{\frac{8}{9}\frac{a^2}{\pi^2}\right\}$ by Mathematica

$$\boxed{\langle x\rangle = \left(\frac{4}{3\pi}\right)^2 a\cos\left[\frac{3}{2}\frac{\hbar\pi^2}{ma^2}t\right]}$$