1. (3.1) Eqtn (3-4): $\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p) e^{ipx/\hbar}$

$$\int_{-\infty}^{\infty} dx\, \psi^*(x) \times \psi(x) = \int dx \left[\frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi^*(p) e^{-ipx/\hbar}\right] \times \left[\frac{1}{\sqrt{2\pi\hbar}} \int dp'\, \phi(p') e^{ip'x/\hbar}\right]$$

(just substitution)

$$= \frac{1}{2\pi\hbar} \int dx \int dp \int dp'\, \phi^*(p) \times e^{i(p'-p)x/\hbar}\, \phi(p')$$

(rearranging)

$$= \frac{1}{2\pi\hbar} \int dp \int dp'\, \phi^*(p)\, \phi(p') \left[\int dx \times e^{i(p'-p)x/\hbar}\right] \qquad (1)$$

(more rearranging)

Note: $\int dx \times e^{i(p'-p)x/\hbar} = \int dx \frac{\hbar}{i} \frac{\partial}{\partial p'} \left[e^{i(p'-p)x/\hbar}\right]$ (def. of derivative)

$$= \frac{\hbar}{i} \frac{\partial}{\partial p'} \left[\int dx\, e^{i(p'-p)x/\hbar}\right] = \frac{\hbar}{i} \frac{\partial}{\partial p'} \left[2\pi\, \delta\left(\tfrac{1}{\hbar}(p'-p)\right)\right] \quad (\text{def. of } \delta\text{-fncn})$$

$$= \frac{\hbar}{i} \frac{\partial}{\partial p'} \left[2\pi\hbar\, \delta(p'-p)\right] \quad (\delta\text{-fncn identity: A-16})$$

$$= (2\pi\hbar) \frac{\hbar}{i} \frac{\partial}{\partial p'} \delta(p'-p)$$

Substitute this back into (1):

$$\int_{-\infty}^{\infty} dx\, \psi^*(x) \times \psi(x) = \frac{\hbar}{i} \int dp \int dp'\, \phi^*(p) \phi(p') \frac{\partial}{\partial p'} \delta(p'-p)$$

$$\overset{\text{Integrate by parts}}{=} \frac{\hbar}{i} \int dp \int dp' \left[\frac{\partial}{\partial p'}\left(\phi^*(p)\phi(p')\delta(p'-p)\right) - \delta(p'-p) \frac{\partial}{\partial p'}\left(\phi^*(p)\phi(p')\right)\right] \qquad (2)$$

(product rule) — term 1 — term 2

term 1: $\int_{-\infty}^{\infty} dp' \frac{\partial}{\partial p'}\left[f(p')\right] = f(p')\Big|_{-\infty}^{\infty}$ (fundamental thm. of calculus)

here, $f(p') = \phi^*(p)\phi(p')\delta(p'-p) = \phi^*(p')\phi(p')$

so: $\int_{-\infty}^{\infty} dp' \frac{\partial}{\partial p'}\left(\phi^*(p)\phi(p')\delta(p'-p)\right) = \phi^*(p')\phi(p')\Big|_{-\infty}^{\infty} = 0$

Since we've postulated that the norm of the wavefunction $\to 0$ at $\pm\infty$.

term 2: $\int_{-\infty}^{\infty} dp'\, \delta(p'-p) \frac{\partial}{\partial p'}\left(\phi^*(p)\phi(p')\right) = \int dp'\, \delta(p'-p) \phi^*(p) \frac{\partial}{\partial p'} \phi(p')$

$$= \phi^*(p) \frac{\partial}{\partial p} \phi(p)$$

Since $\int \delta(p'-p) f(p') dp' = f(p)$, def. of δ-fncn.