UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 115A
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Winter 1996

Practice

FINAL EXAM (con't.)

4. A particle of mass m is bound in a one-dimensional potential $V = \frac{1}{2}kx^2$. The particle is in the ground state.

   (a) What is the probability $dP(x)$ that the particle is found at the location x within dx?

from the equation page

$$dP(x) = |u(x)|^2 dx$$

$$u_0(x) = \sqrt{\frac{\beta}{\sqrt{\pi}}}\, e^{-\frac{\beta^2 x^2}{2}}$$

$$dP(x) = \frac{\beta}{\sqrt{\pi}}\, e^{-\beta^2 x^2} \qquad \beta = \sqrt{\frac{m\omega}{\hbar}}$$

   (b) What is the value of $\langle X^2 \rangle$?

$$X = \frac{A + A^\dagger}{\sqrt{2}\,\beta}$$

$$\langle u_0 | X^2 | u_0 \rangle = \frac{1}{2\beta^2} \langle u_0 | A^2 + AA^\dagger + A^\dagger A + A^{\dagger 2} | u_0 \rangle$$

$$A|u_0\rangle = 0, \quad A^\dagger |u_0\rangle = |u_1\rangle, \quad A|u_1\rangle = |u_0\rangle$$

only surviving term is $\frac{1}{2\beta^2}\langle u_0 | AA^\dagger | u_0 \rangle$

$$= \frac{1}{2\beta^2}\langle u_0 | A | u_1 \rangle = \frac{1}{2\beta^2}$$

$$\langle X^2 \rangle = \frac{1}{2\beta^2}$$