UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 115A
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Practice

FINAL EXAM (con't.)

3. A particle of mass m is in a 2-dimensional infinite square well; $V = 0$ $0 < x < a$ and $0 < y < a$, $V = \infty$ for $x < 0$ or $x > a$ or $y < 0$ or $y > a$.

(a) Find the lowest energy eigenfunction $\phi(x, y)$.

Sch. eqn $-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] u(x,y) = E\,u(x,y)$

Separate variables $u(x,y) = \phi(x)\chi(y)$

$\phi(x) = 0$ at $x = 0, a$, $\chi(y) = 0$ at $y = 0, a$

try $u_{n_x,n_y}(x,y) = \left(\sqrt{\frac{2}{a}}\right)^2 \sin\frac{n_x \pi x}{a} \sin\frac{n_y \pi y}{a}$

into sch. eqn.

$$\left[+\frac{\hbar^2}{2m}\right]\left[\left(\frac{n_x\pi}{a}\right)^2 + \left(\frac{n_y\pi}{a}\right)^2\right] u_{n_x,n_y} = E_{n_x,n_y}\, u_{n_x,n_y}(x,y)$$

$u_{1,1} = \frac{2}{a}\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}$

(b) What are the possible energies of the system?

$$E_{n_x,n_y} = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2\left(n_x^2 + n_y^2\right)$$