Question: On page 39 of the professor's notes, he said the elements of U were $\langle i'|j\rangle$ instead of $\langle i|j'\rangle$. Anyone who followed that example would have done Problem 3 of homework 3 wrong.

When Prof. Nelson talks about transforming matrix elements on page 40, he gets the boxed equation,

$$\Omega_{il} = \sum_{jk} U^\dagger_{ij} \Omega'_{jk} U_{kl}$$

which transforms the matrix elements of the "primed" representation of Ω into the matrix elements in the unprimed representation. From the line above that, $U_{kl} = \langle k'|l\rangle$ and $U^\dagger_{ij} = \langle i|j'\rangle$ is its adjoint.

The way I've been deriving these things in my solutions is by inserting a decomposition of unity. If I do that for the components of a vector,

$$v'_{j'} = \langle j'|v\rangle = \sum_i \langle j'|i\rangle\langle i|v\rangle = \sum_i \langle j'|i\rangle v_i$$

Comparing with the previous paragraph, I can identify the braket $\langle j'|i\rangle = U_{ji}$ and write

$$\langle j|v\rangle = \sum_i U_{ji}\langle i|v\rangle$$

This is the same thing that Prof. Nelson has on page 39–there's no mistake.
I think the confusing thing is that in Prof. Nelson's notes, U is the unitary matrix transformation from the initial primed basis to the new unprimed basis. Just for this discussion, call this the "forward" direction.
If, instead, you call the initial basis the unprimed basis and want to transform to the primed basis, you're going in "reverse" instead of forward. The correct matrix to perform the reverse transformation is the inverse of the transformation in the forward direction, U^{-1}. But since $U \doteq U_{ji} = \langle j'|i\rangle$ is unitary, $U^{-1} = U^\dagger \doteq U^\dagger_{ij} = \langle i|j'\rangle$. So, in summary, pages 38-40 and thereabouts of Nelson's

notes are right, provided you use these definitions:

$$U_{ji} = \langle j'|i\rangle$$
$$U^\dagger_{ij} = \langle i|j'\rangle$$
$$\overbrace{\langle i|\underset{\sim}{\Omega}|l\rangle}^{\Omega_{il}} = \sum_{jk} \underbrace{\langle i|j'\rangle}_{U^\dagger_{ij}} \overbrace{\langle j'|\underset{\sim}{\Omega}|k'\rangle}^{\Omega'_{jk}} \underbrace{\langle k'|l\rangle}_{U_{kl}}$$
$$\overbrace{\langle i'|\underset{\sim}{\Omega}|l'\rangle}^{\Omega'_{il}} = \sum_{jk} \underbrace{\langle i'|j\rangle}_{U_{ij}} \overbrace{\langle j|\underset{\sim}{\Omega}|k\rangle}^{\Omega_{jk}} \underbrace{\langle k|l'\rangle}_{U^\dagger_{kl}}$$
$$\overbrace{\langle i|v\rangle}^{v_i} = \sum_{j} \underbrace{\langle i|j'\rangle}_{U^\dagger_{ij}} \overbrace{\langle j'|v\rangle}^{v'_j}$$
$$\overbrace{\langle j'|v\rangle}^{v'_j} = \sum_{i} \underbrace{\langle j'|i\rangle}_{U_{ji}} \overbrace{\langle i|v\rangle}^{v_i}$$

If instead you want to think of the transformation from the unprimed to primed basis as the forward direction, which is the way I think of it in the homework solutions, then the thing Nelson calls U is what you would call $U^\dagger$ and what he calls $U^\dagger$ you would call U. Either way of thinking about it is fine, as long as your definitions can reproduce the equations,

$$\langle i|\underset{\sim}{\Omega}|l\rangle = \sum_{jk} \langle i|j'\rangle\langle j'|\underset{\sim}{\Omega}|k'\rangle\langle k'|l\rangle$$
$$\langle i'|\underset{\sim}{\Omega}|l'\rangle = \sum_{jk} \langle i'|j\rangle\langle j|\underset{\sim}{\Omega}|k\rangle\langle k|l'\rangle$$
$$\langle i|v\rangle = \sum_{j} \langle i|j'\rangle\langle j'|v\rangle$$
$$\langle j'|v\rangle = \sum_{i} \langle j'|i\rangle\langle i|v\rangle$$