Physics 115A Third Problem Set

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1. The three kets $|1\rangle$, $|2\rangle$, and $|3\rangle$ are represented, in a particular basis as:

$$|1\rangle \doteq \begin{bmatrix} 1/2 \\ \sqrt{3}/2\sqrt{2} \\ \sqrt{3}/2\sqrt{2} \end{bmatrix}, \; |2\rangle \doteq \begin{bmatrix} -\sqrt{3}/2 \\ 1/2\sqrt{2} \\ 1/2\sqrt{2} \end{bmatrix}, \; |3\rangle \doteq \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}.$$

   (a) Find the matrices that represent $|1\rangle\langle 1|$, $|2\rangle\langle 2|$, $|1\rangle\langle 1| + |2\rangle\langle 2|$, and $|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|$.

   (b) Use the matrix representation to evaluate the action of $|1\rangle\langle 1| + |2\rangle\langle 2|$ on $|3\rangle$, and interpret the result.

   (c) Evaluate and interpret the trace of $|1\rangle\langle 1| + |2\rangle\langle 2|$.

2. Use an insertion of the 'decomposition of unity,' which is the relationship $\sum_{k=1}^{n} |k\rangle\langle k| = \mathbf{I}$, to derive the 'matrix multiplication' relationship between the matrix elements of a sequence of two linear operators, $\mathbf{\Lambda\Omega}$, with matrix elements in the basis of $\langle i|\mathbf{\Lambda\Omega}|j\rangle$, and the matrix elements of the individual operators $\langle i|\mathbf{\Lambda}|k\rangle$ and $\langle k|\mathbf{\Omega}|j\rangle$.

3. In class, and on page 22 of your text, the representation of the rotation operator $R(\frac{1}{2}\pi\mathbf{i})$ in the 'standard' basis was derived. Now, for a challenge, represent $R(\frac{1}{2}\pi\mathbf{i})$ in the basis described by the three kets given in problem 1 (relabel those as $|1'\rangle$, $|2'\rangle$, and $|3'\rangle$). Assume that the three components of each ket in problem 1 are, respectively, the x, y, and z components of the new basis kets. Make a drawing showing what those kets in problem 1 look like in the xyz coordinate system. You don't need to use the drawing to obtain the new representation of $R(\frac{1}{2}\pi\mathbf{i})$, however; it is sufficient to put together the unitary transformation needed, as described in class, and to apply them.

4. Exercise 1.6.3 on page 28 of your text.

5. Exercise 1.6.6 on page 29 of your text.